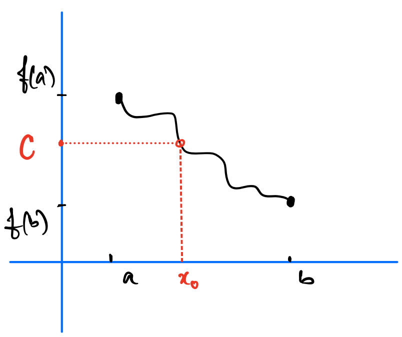
More on Continuity

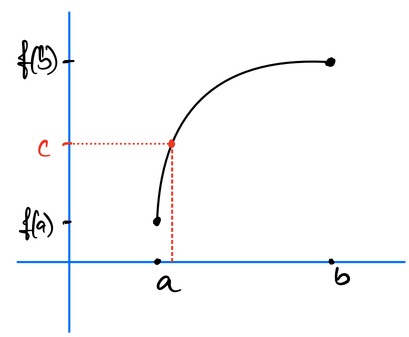
Intermediate Value Theorem:

If f is continuous on $[a,b]$ [ie, you can draw a continuous curve joining $(a, f(a))$ & $(b, f(b))$], then if c is any number between $f(a)$ & $f(b)$.

Then there is atleast one x_0 between a & b with

$$f(x_0) = c.$$





Application: $f(x) = \cos x - x \rightsquigarrow$ Continuous on $[0, \frac{\pi}{2}]$.

Now, $f(0) = 1$, $f(\pi/2) = -\frac{\pi}{2}$

Now, $-\frac{\pi}{2} \leq 0 \leq 1$

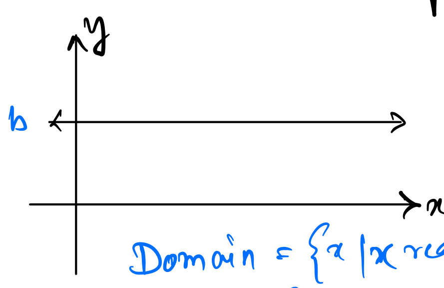
So by IVT, there is some θ between 0 & $\pi/2$ such that

$f(\theta) = 0 \Rightarrow \cos\theta - \theta = 0$

$\Rightarrow \cos\theta = \theta$

Overview for Midterm

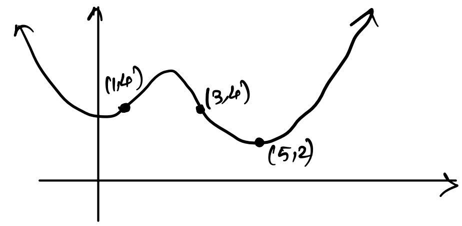
① Domain & Range:



Domain = $\{x \mid x \text{ real}\}$

Range = $\{y \mid y = b\}$

Find the range of



$\{y \mid y \geq 2\}$

④ What can you say about the range of

$$f(x) = 4(x+3)^2 - 7, \quad x \text{ is in } (-\infty, \infty)$$

② Logarithms:

Simplify (i) $\log_2 \left(\frac{4x^3}{y^2}\right)$

(ii) $\log_{10} (x^2+1)$

(iii) $\log_{10} ((x+1)^2)$

Solve for x: $\log_{10}(x+3) = \log_{10}(4) - \log_{10}(x)$

Note: Domain for $\log x$ is $x > 0$

③ Limits

Find the limit, if exists:

$$\lim_{x\to 3} \frac{x^2-3x}{\sqrt{x+1}-2}$$

Note: Direct Substitutions leads us to $\frac{0}{0}$. So we have to work more.

$$\lim_{x\to 3} \frac{x(x-3)}{\sqrt{x+1}-2}$$

$$= \lim_{x\to 3} \frac{x(x-3)}{\sqrt{x+1}-2} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$$

$$= \lim_{x\to 3} \frac{x(x-3)\cdot(\sqrt{x+1}+2)}{(x+1)-2^2}$$

$$= \lim_{x\to 3} \frac{x\cancel{(x-3)}(\sqrt{x+1}+2)}{\cancel{(x-3)}}$$

$$= \lim_{x\to 3} x(\sqrt{x+1}+2)$$

$$= 3(\sqrt{3+1}+2) = 3\cdot(2+2) = 12.$$